Mathematics: Advanced Placement Calculus AB and BC Honors

Special Content Standard #4 Advanced Placement Mathematics

Area #4A Functions, Graphs, Limits

Area #4B Derivatives

Area #4C Integrals

Area #4D Polynomial Approximations and Series

Description

This is a two-semester college level class that covers limits, derivatives, integrals, series and sequences, vectors, parametric and polar equations. In addition, techniques in solving problems involving derivatives, integrals, and series and sequences will be addressed. Applications of calculus in the areas of physics, business and economics, and other fields are also discussed and solved. All of the topics evaluated by the Advanced Placement BC Calculus Test will be addressed. Graphing calculator technology (TI-83 Plus and TI-89) will be regularly used. Students may purchase their own books. Students may also take the AP Calculus test in May. The cost of the test is approximately $86. Students may also choose to receive eight semester hours of credit from the University of Colorado at Denver.

Time Allocation

90 minutes daily for AP Calculus AB first semester and 90 minutes daily for AP Calculus BC second semester

Texts/References

Calculus: Graphical, Numerical, Algebraic, Scott Foresman Addison Wesley, 2007

Assessments

One or two tests for each chapter
Quizzes
Activities
Three midterm exams
Final
AP Calculus test given in May

Grades and Achievement Levels

Grades are based on completion of assignments and quality of work.

Expectations: Mathematics Advanced Placement Calculus AB and BC Honors

Area #4A
Functions, Graphs, Limits

1. Analysis of graphs: With the aid of technology, graphs or functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus to predict and to explain the observed local and global behavior of a function.
2. Limits of functions, including one-sided limits
   a. An intuitive understanding of the limiting process
   b. Calculating limits using algebra
   c. Estimating limits from graphs or tables of graphs
3. Asymptotic and unbounded behavior
   a. Understanding asymptotes in terms of graphical behavior
   b. Describing asymptotic behavior in terms of limits involving infinity
   c. Comparing relative magnitudes of functions and their rates of change (exponential vs polynomial vs logarithmic)

Area #4B
Derivatives

1. Concept of the derivative
   a. Derivative presented geometrically, numerically and analytically
   b. Derivative interpreted as an instantaneous rate of change
   c. Derivative defined as the limit of the difference quotient
   d. Relationship between differentiability and continuity
2. Derivative at a point
   a. Slope of a curve at a point. Examples are emphasized, including points at which there are vertical tangents and points at which there are no tangents.
   b. Tangent line to a curve at a point and local linear approximation
   c. Instantaneous rate of change as the limit of average rate of change
   d. Approximate rate of change from graphs and tables of values
3. Derivatives as a function
   a. Corresponding characteristics of the graphs of f and f'
   b. Relationship between the increasing and decreasing behavior of f and the sign of f'
   c. The Mean Value Theorem and its geometric consequences
   d. Equations involving derivatives (verbal descriptions are translated into equations involving derivates and vice versa)

Area #4C
Integrals

1. Interpretations and properties of definite integrals
   a. Definite integral as a limit of Riemann sums over equal subdivisions
   b. Definite integral of the rate of change of a quantity over an interval interpreted as the change of the quantity over the interval:

   $$\int_a^b f'(x)dx = f(b) - f(a)$$

   c. Basic properties of definite integrals (additivity and linearity)
   d. *Appropriate integrals are used in a variety of applications to model physical, biological, or economic situation. Although only a sampling of applications can be included in any specific course, students should be able to adapt their knowledge and techniques to solve other similar application problems. Whatever applications are chosen, the emphasis is on using the integral of a rate of change to give accumulated change or using the method of setting up an approximating

Area #4D
Polynomial Approximations and Series

1. *Concept of series (defined as a sequence of partial sums and convergence is defined in terms of the limit of the sequence of partial sums)
2. *Series of constants
   a. Motivating examples including decimal expansion
   b. Geometric series with applications
   c. The harmonic series
   d. Alternating series with error bound
   e. Terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series
   f. The ratio test for convergence and divergence
   g. Comparing series to test for convergence or divergence
3. *Taylor Series
   a. Taylor polynomial approximation with graphical demonstration of convergence. (Example, viewing graphs of various Taylor polynomials of the

Expectations: Mathematics Advanced Placement Calculus AB and BC Honors continued

Area #4A
Functions, Graphs, Limits

4. Continuity as a property of functions
   a. An intuitive understanding of continuity; close values of the domain lead to close values of the range
   b. Understanding continuity in terms of limits
   c. Geometric understanding of graphs of continuous function (Intermediate Value Theorem and Extreme Value Theorem)
5. *Parametric, polar and vector functions: The analysis of planar curves includes those given in parametric form, polar form, and vector form

Area #4B
Derivatives

4. Second Derivatives
   a. Corresponding characteristics of the graphs of f, f', f''
   b. Relationship between the concavity of f and the sign of f''
   c. Points of inflection as places where concavity changes
5. Application of derivatives
   a. Analysis of curves, including notions of monotonicity and concavity
   b. *Analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration vectors
   c. Optimization, both absolute (global) and relative (local) extrema
   d. Modeling rates of change including related rates problems
   e. Use of implicit differentiation to find the derivative of an inverse function
   f. Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration
   g. *Geometric interpretation of differential equations via slope fields and the relationship between slope fields and derivative of implicitly defined functions

Area #4C
Integrals

Riemann sum and representing its limit as a definite integral. To provide a common foundation, specific applications should include finding the area of a region (including a region bounded by polar curves), the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, the length of a curve (including a curve given in parametric form), and accumulated change from rate of change.

2. Fundamental Theorem of Calculus
   a. Use of the Fundamental Theorem to evaluate definite integrals
   b. Use of the Fundamental Theorem to represent a particular antiderivative and the analytical and graphical analysis of functions so defined
3. Techniques of antidifferentiation
   a. Antiderivatives following directly from derivative of basic functions
   b. *Antiderivatives by substitution of variables (including change of limits for definite integrals, parts, and

Area #4D
Polynomial Approximations and Series

of the since function approximating the sine curve)

b. Maclaurin series and the generl Taylor series centered at x=a
c. Maclaurin series for the functions $e^{x,}$ sin x, cos x, and 1/1-x
d. Formal manipulation of Taylor series and shortcuts to series
e. Functions defined by power series
f. Radius and interval of convergence of power series
g. Lagrange error bound for Taylor polynomials

Expectations: Mathematics Advanced Placement Calculus AB and BC Honors continued

Area #4A **Functions, Graphs, Limits**	**Area #4B** **Derivatives**	**Area #4C** **Integrals**	**Area #4D** **Polynomial Approximations and Series**
	h. *Numerical solution of differential equations using Euler's method i. *L'Hopital's Rule and its use in determining convergence of improper integral and series 6. Computation of derivatives a. Knowledge of derivative of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions b. Basic rules for the derivative of sums, products, and quotients of functions c. Chain rule and implicit differentiation d. *Derivatives of parametric, polar, and vector functions	Simple partial fractions (nonrepeating linear factors only) c. *Improper integrals (as limits of definite integrals) 4. Numerical approximations to definite integrals. Use of Riemann and trapezoidal sums to approximate definite integrals of functions represented algebraically, geometrically and by tables of values	